Problem 12-28

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)$ ft/s^2 and $a_B = (12t^2 - 8)$ ft/s^2, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

Solution

Because the particles start from rest at the origin, the initial conditions are $s_A(0) = 0$, $v_A(0) = 0$, $s_B(0) = 0$, and $v_B(0) = 0$. Use the given accelerations to get the velocities.

$$a_A = \frac{dv_A}{dt} = 6t - 3 \qquad\qquad a_B = \frac{dv_B}{dt} = 12t^2 - 8$$

Integrate both sides with respect to t.

$$v_A(t) = 3t^2 - 3t + C_1 \qquad\qquad v_B(t) = 4t^3 - 8t + C_2$$

Use the initial conditions to determine C_1 and C_2.

$$v_A(0) = C_1 = 0 \qquad\qquad v_B(0) = C_2 = 0$$

As a result, the velocities (in feet per second) are

$$v_A(t) = 3t^2 - 3t \qquad\qquad v_B(t) = 4t^3 - 8t.$$

With them, the positions can be obtained.

$$\frac{ds_A}{dt} = 3t^2 - 3t \qquad\qquad \frac{ds_B}{dt} = 4t^3 - 8t$$

Integrate both sides with respect to t.

$$s_A(t) = t^3 - \frac{3}{2}t^2 + C_3 \qquad\qquad s_B(t) = t^4 - 4t^2 + C_4$$

Use the initial conditions to determine C_3 and C_4.

$$s_A(0) = C_3 = 0 \qquad\qquad s_B(0) = C_4 = 0$$

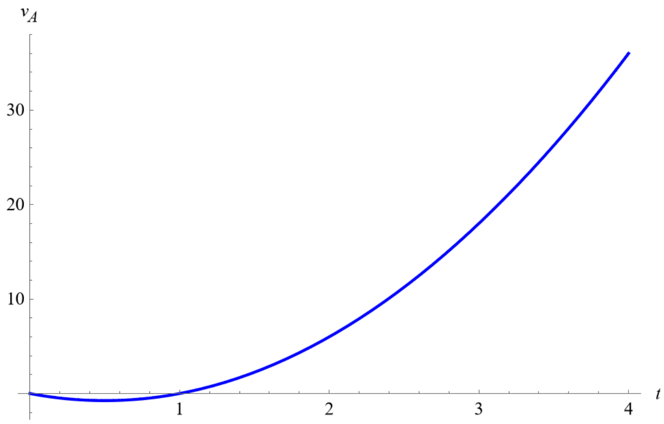
As a result, the positions (in feet) are

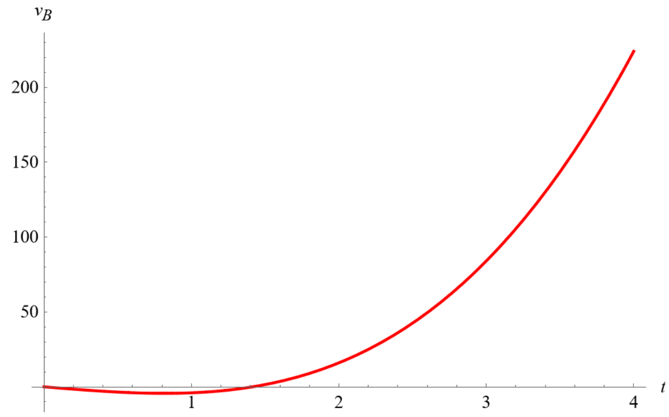
$$s_A(t) = t^3 - \frac{3}{2}t^2 \qquad\qquad s_B(t) = t^4 - 4t^2.$$

The distance between the particles when $t = 4$ s is $|s_A(4) - s_B(4)| = 152$ ft. Now calculate the total distance that each particle travels in $t = 4$ s by integrating their respective speeds.

$$(s_A)_{\text{total}} = \int_0^4 |v_A(t)|\, dt \qquad\qquad (s_B)_{\text{total}} = \int_0^4 |v_B(t)|\, dt$$

Below is a plot of the velocities versus time to see where they're positive and negative.





Find where the zeros of $v_A(t)$ and $v_B(t)$ are.

$$v_A(t) = 3t^2 - 3t = 0$$
$$3t(t-1) = 0$$
$$t = \{0, 1\}$$

$$v_B(t) = 4t^3 - 8t = 0$$
$$4t(t^2 - 2) = 0$$
$$t = \{-\sqrt{2}, 0, \sqrt{2}\}$$

Therefore,

$$\begin{aligned}(s_A)_{\text{total}} &= \int_0^4 |v_A(t)|\,dt \\ &= \int_0^1 (-3t^2 + 3t)\,dt + \int_1^4 (3t^2 - 3t)\,dt \\ &= \left(-t^3 + \frac{3}{2}t^2\right)\Big|_0^1 + \left(t^3 - \frac{3}{2}t^2\right)\Big|_1^4 \\ &= \left(\frac{1}{2}\right) + \left(\frac{81}{2}\right) \\ &= 41\end{aligned}$$

$$\begin{aligned}(s_B)_{\text{total}} &= \int_0^4 |v_B(t)|\,dt \\ &= \int_0^{\sqrt{2}} (-4t^3 + 8t)\,dt + \int_{\sqrt{2}}^4 (4t^3 - 8t)\,dt \\ &= (-t^4 + 4t^2)\Big|_0^{\sqrt{2}} + (t^4 - 4t^2)\Big|_{\sqrt{2}}^4 \\ &= (4) + (196) \\ &= 200\end{aligned}$$

Therefore, particle A travels a total distance of 41 feet, and particle B travels a total distance of 200 feet.